Advanced Technologies for Hypersonic Propulsive, Energetic and Reusable Platforms
Technical Conference and Exposition
30 July 2021

QUANTUM MACHINE LEARNING FOR SOLVING LINEAR SYSTEMS

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ABSTRACT

As the computing power needed to handle high level problems outpaces the abilities of traditional computers, programmers have looked into an entirely new system of computation to meet their needs. Unlike the traditional binary system of 1s and 0s used in classical computing, quantum computing exploits the properties of quantum states to perform computations substantially faster than their classical counterparts. In fields such as machine learning, where the computational power needed to accurately render and solve real-life physical problems is often prohibitively high, the efficiency afforded by quantum computers has the potential to revolutionize the field and enable researchers to model scenarios with unprecedented accuracy.

Here a method of solving linear systems through quantum machine learning is explored.

NOMENCLATURE

QML	Quantum Machine Learning
TF	Tensorflow
SF	StrawberryFields
Qubit	Quantum bit
VQLS	Variational Quantum Linear Solver

1. INTRODUCTION

Interest in the field of quantum computing has increased substantially in recent years, with both public and private entities investing in the technology. Companies such as IBM, Amazon and Google have all begun their own research in the field in hopes of competing in the emerging market.

The most commonly used model is known as the quantum circuit model and employs a quantum analogue to the classical bit- quantum bits, or qubits- for performing computations. The most basic unit of information in a quantum system, a qubit is a two-state QM system that as such embodies characteristics such as superposition. Thus, unlike the binary bit which is either a 1 or a 0, the qubit is able to, like Schrodinger's cat, be both 1 and 0 at the same time. This enables them to encode more information than the classical bit, up to the equivalent of two bits per qubit.

The increased computational efficiency of quantum computers is of particular interest to those studying machine learning for solving linear systems, where physical fidelity is often achieved at the cost

of computational efficiency [1]. Though solutions such as a hybrid machine-learning/physics informed model have been proposed, another possible answer would be to increase the computational efficiency of the machines themselves.

Whereas a classical computer's runtime in solving a linear system matrix is proportional to the number of variables N, a quantum computer's runtime would be proportional to the log of the number of variables, log(N) [5]. As the number of variables climbs into the tens or hundreds of thousands, as often happens with physically representative linear systems, this difference in runtime becomes quite substantial.

Combining the traditional methods of machine learning with the computational power of quantum computers enables researchers to solve linear systems with a degree of complexity as yet unmatched by any other method.

2. MATERIALS AND METHODS

Xanadu is a quantum technology company that makes its software available to all with the aim of increasing the general accessibility of quantum computers [2]. Users are given access to the company's Quantum Cloud which allows them to make use of the company's near-term quantum devices.

The company provides two crucial pieces of software that prove useful for the task at hand: PennyLane, a library dedicated to quantum machine learning, and StrawberryFields, a plugin that optimizes PennyLane for use with Tensorflow, an end-to-end platform for building machine learning models.

The Variational Quantum Linear Solver (VQLS), outlined in a tutorial on the Xanadu website, employs quantum machine learning to solve linear systems of the form $A\vec{x} = \vec{b}$, where A is a 2^n*2^n matrix and $\vec{b}$ is a vector [3]. The matrix A must be expressed as a linear combination of unitaries, achievable using NumPy, and $\vec{b}$ must be prepared into the quantum state $|b\rangle$ using a short depth quantum circuit U of the form $|b\rangle = U|0\rangle$.

Using a typical machine learning approach, a loss function is constructed within the algorithm and minimized until its value falls below a certain threshold. The values used to achieve this are then logged as the values for $\vec{x}$. The system then returns an expectation value $\langle x|n\rangle$.

We used the VQLS to solve a 15x15 linear system detailing the forces on a metal beam. The A matrix and $\vec{b}$ vector were converted into usable inputs and run through the algorithm.

3. RESULTS AND DISCUSSION

The VQLS was able to successfully output a solution to the 15x15 linear system as well as depict the decreasing cost function over each successive approximation. For a 15x15 linear system the runtime is estimated to be on the order of 100x shorter than that of a classical computer.

The gap between quantum machine learning efficiency and classical machine learning efficiency is expected to only get larger as systems grow larger and more complex. While the VQLS offered an efficient way to solve large linear systems, the necessity of converting the matrix A to a linear combination of unitaries and the vector b into a quantum state required the use of separate, classical computing software such as NumPy. For larger systems of equations, the computational cost of these operations would be prohibitively high. For the software to be truly viable up to a size of "$2^{50} * 2^{50}$"[3], a method for decomposing the matrix and vector from their original format into the required input format would need to be embedded efficiently into the quantum algorithm.

Aside from this relatively minor issue, the VQLS offers a promising alternative to the classical machine learning model for linear systems, and contributes to the growing body of evidence that quantum machine learning may be the future of, if not all machine learning, at least that which concerns the solving of complex mathematical problems.

4. CONCLUSION

As the problems physicists and engineers are tasked with solving grow ever more complex, the ability of classical computers to efficiently solve these problems rapidly approaches its limit. With companies such as Google, Amazon, IBM, and Microsoft pouring resources into their own quantum computing research, it's become clear that technological market giants have caught wind of these limitations as well, and are seeking a suitable alternative in the promise of quantum computing.

While it may be a number of years before quantum computers are widely available to a consumer base, a growing body of research indicates that quantum computing may be more than a passing fad. In areas of research increasingly burdened by the computational limits of classical computers, such as machine learning, we may see quantum computers supplant classical computers entirely within the next few decades.

ACKNOWLEDGEMENTS

This work is supported by the National Science Foundation and the U.S. Department of Defense in partnership with the University of Central Florida HYPER program.

Special thanks to my advisor Dr. Felipe Viana and graduate student Kajetan Fricke for their guidance in my research, as well as Robert Burke, Dr. Gordon, and Dr. Kaufman for organizing the HYPER program here at UCF.

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