DYNAMIC MODE DECOMPOSITION TO ANALYZE DETONATION WAVE FREQUENCIES

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Introduction to Detonation Waves

Most chemical energy conversions with jet/rocket propulsion comes from combustion-based deflagration [1], which has been demonstrated to be inefficient compared to detonation-based deflagration because it exhausts massive energy from the combustion process [2]. One type of engine that utilizes detonation-based deflagration is the rotating detonation engine, which uses detonation in a manner such that detonation waves are created and rotate around the annulus of the rotating detonation engine (RDE).

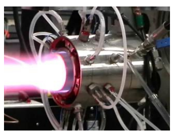

Figure 1: Rotating Detonation Engine [3]

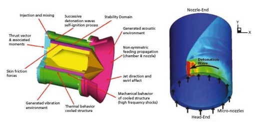

Figure 2: Schematic of Detonation Waves in RDE [4]

As these detonation waves rotate around the annulus, one can find the frequencies of which these waves are traveling at. One paper found them through image correction, annulus location determination, cartesian mesh integration, and frequency domain analysis [5]. This research tries to accomplish this same feat but with dynamic mode decomposition (DMD).

Dynamic Mode Decomposition

$$\mathbf{X} = \begin{bmatrix} | & | & & | \\ \mathbf{x}_1 & \mathbf{x}_2 & \cdots & \mathbf{x}_m \\ | & | & & | \end{bmatrix}$$

$$\mathbf{X}' = \begin{bmatrix} | & | & & | \\ \mathbf{x}_2 & \mathbf{x}_3 & \cdots & \mathbf{x}_{m+1} \\ | & | & & | \end{bmatrix}$$

Figure 3: X and X′ Matrix of m Length and n Height [6]

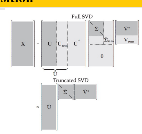

Figure 4: Singular Value Decomposition of X Matrix [6]

Dynamic mode decomposition (DMD) is a way to identify spatio-temporal coherent structures from high-dimensional data using proper orthogonal decomposition and singular value decomposition [6]. These spatio-temporal coherent structures are in the form of matrices which are denoted as $\mathbf{X}$ and $\mathbf{X'}$ (Equations/Definitions 1 and 2). These matrices have dimensions of m by n for which m represents time and n represents space. Note that, in Figure 3, $\mathbf{X}$ and $\mathbf{X'}$ are demonstrated to be related the fact that $\mathbf{X'}$ is one time step ahead of $\mathbf{X}$.

One main goal of DMD is to linearly approximate $\mathbf{X}$ and $\mathbf{X'}$ with a matrix $\mathbf{A}$ (Equation 3). The process starts with finding the singular value decomposition (SVD) of the $\mathbf{X}$ matrix, which creates the matrices $\mathbf{U}$, $\mathbf{\Sigma}$, and $\mathbf{V}$. Equations 5, 6, and 7 display the elements of these matrices within the dimensions of m and n. We use Equation 9 to find the lowest r value that fits that criterion. This truncates matrices found by SVD to the elements demonstrated by equations/definitions 9-11. The rest of the equations show how to find the ranked matrix A, the eigenvalues/eigenvectors of A, and the DMD modes.

Equation/ Definition #	Equation/ Definition Name	Equation/Definition
1	Elements of X Matrix	$\mathbf{X} \in \mathbb{C}^{n\times m}$
2	Elements of X′ Matrix	$\mathbf{X}' \in \mathbb{C}^{n\times m}$
3	Linear Relation between X and X′ Matrices	$\mathbf{X}' \approx \mathbf{AX}$
4	SVD of X	$\mathbf{X} \approx \mathbf{U\Sigma V}^*$
5	Elements of $\mathbf{U}$ Matrix	$\mathbf{U} \in \mathbb{C}^{n\times n}$
6	Elements of $\mathbf{\Sigma}$ Matrix	$\mathbf{\Sigma} \in \mathbb{R}^{n\times m}$
7	Elements of $\mathbf{V}$ Matrix	$\mathbf{V} \in \mathbb{C}^{m\times m}$
8	Ranked $\mathbf{X}$ Decomposition denoted as $\tilde{\mathbf{X}}$	$\mathbf{X} \approx \mathbf{U\Sigma V}^*$ (ranked SVD) $\tilde{\mathbf{X}} \approx \tilde{\mathbf{U}}\tilde{\mathbf{\Sigma}}\tilde{\mathbf{V}}^*$
9	Criterion to find r	$.999 \geq \frac{\sum_{k=1}^{r}\sigma_k}{\sum_{k=1}^{m}\sigma_k}$
10	Elements of $\tilde{\mathbf{U}}$ Matrix	$\tilde{\mathbf{U}} \in \mathbb{C}^{n\times r}$
11	Elements of $\tilde{\mathbf{\Sigma}}$ Matrix	$\tilde{\mathbf{\Sigma}} \in \mathbb{C}^{r\times r}$
12	Elements of $\tilde{\mathbf{V}}$ Matrix	$\tilde{\mathbf{V}} \in \mathbb{C}^{m\times r}$
13	Matrix $\mathbf{A}$	$\mathbf{A} = \mathbf{X}'\tilde{\mathbf{V}}\tilde{\mathbf{\Sigma}}^{-1}\tilde{\mathbf{U}}^*$
14	Ranked Matrix $\mathbf{A}$ (denoted as $\tilde{\mathbf{A}}$)	$\tilde{\mathbf{A}} = \tilde{\mathbf{U}}^*\mathbf{X}'\tilde{\mathbf{V}}\tilde{\mathbf{\Sigma}}^{-1}$
15	Spectral Decomposition of $\tilde{\mathbf{A}}$	$\tilde{\mathbf{A}}\mathbf{W} = \mathbf{W\Lambda}$
16	DMD Modes	$\mathbf{\Phi} = \mathbf{X}'\tilde{\mathbf{V}}\tilde{\mathbf{\Sigma}}^{-1}\mathbf{W}$

Photron Fastcam Viewer/Tiffs

The detonation waves are collected in the form of tiff files from Photron Fastcam Viewer, a software that allows users to analyze high-speed images and export them for future use.

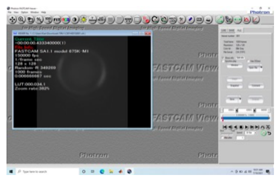

Figure 5: Photron Fastcam Viewer With Tiff File of Detonation Wave

DMD Code

Figure 7 shows a barebone example of a DMD Code which takes the SVD of the X Matrix, ranks the $\mathbf{U}$, $\mathbf{\Sigma}$, and $\mathbf{V}$ matrices, and finds the $\mathbf{A}$ matrix and its eigenvalues/eigenvectors. Figure 8 displays Equation/Definition 9 in code form.

```
[U,Sigma,V] = svd(X,'econ');        % Step 1
Ur = U(:,1:r);
Sigmar = Sigma(1:r,1:r);
Vr = V(:,1:r);

Atilde = Ur'*Xprime*Vr/Sigmar;      % Step 2
[W,Lambda] = eig(Atilde);           % Step 3

Phi = Xprime*(Vr/Sigmar)*W;         % Step 4
```

Figure 7: DMD Code [6]

```
cdS = cumsum(diag(S))./sum(diag(S));  % Cumulative energy
r90 = min(find(cdS>0.90));  % Find r to capture 90% energy
```

Figure 8: Criterion to find r (r90 is the same as r in Figure 8) [6]

Note that the diagonals from the $\mathbf{\Sigma}$ matrix are used to find the r value and that the elements in the $\mathbf{V}$ matrix are used for the frequency analysis.

Uploading Tiff Files onto MATLAB

These tiff files were then uploaded to MATLAB and were collected and remade into the $\mathbf{X}$ and $\mathbf{X'}$ matrices by making the images into long, skinny matrices (spatial data) and lining all of them up (temporal data). Figure 6 shows an example code of this process.

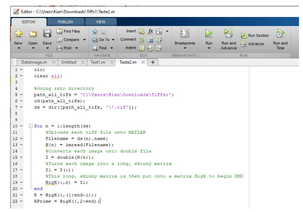

Figure 6: MATLAB Code to Upload Tiff Files and create Matrices for DMD

Frequency Analysis

Frequency analysis means trying to find the frequency domain of a signal's time domain. We use Discrete Fourier Transform (DFT) to find the approximate frequency domain. Fast Fourier Transform (FFT) is an algorithm to try to find the DFT of samples of a signal.

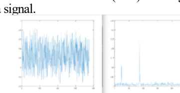

Figure 9: FFT of a Random Signal

Furthermore, one can use Welch's Method to find a version of the FFT with less interference (noise) by doing DFT averaging over sections of the signal that overlap one another.

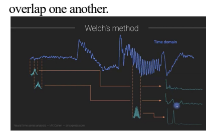

Figure 10: FFT Averaging for Welch's Method [7]

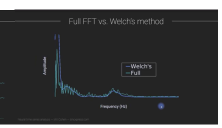

Figure 11: Welch's Method versus FFT Frequency Domains

Exhaust Analysis

While taking the frequency of detonation waves using tiff files that display detonation waves such as the example in Figure 5 is sufficient to show that DMD works for this type of application, the real analysis comes into play when finding the frequencies of detonation waves from views from the exhaust of the RDE. Figures 12-15 show activity from the RDE and one can extract the frequencies through DMD analysis with tiff files of these images. However, the detonation waves are not visible to detect. Therefore, one must crop the images to find the frequencies of the detonation waves. Furthermore, if the full images (Figures 12 and 14) are used, then there will be frequencies that are the result of noise interference rather than just the detonation waves.

Figure 12: 1st Exhaust Full Picture

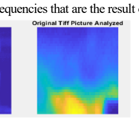

Figure 13: 1st Exhaust Cropped Picture

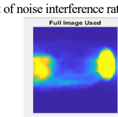

Figure 14: 2nd Exhaust Full Picture

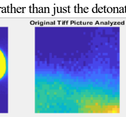

Figure 15: 2nd Exhaust Cropped Picture

Results

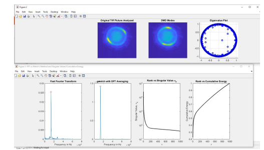

Figure 16: Frequency of Detonation Waves Tiff Set 1 by Research Mentors

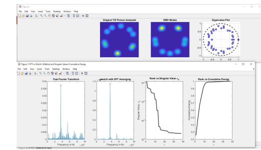

Figure 17: Frequency of Detonation Waves Tiff Set 2 by Post Doc from Previous Experiences

These first two DMD analyses are of tiff files of direct detonation waves, as seen in the pictures within the figures labeled *Original Tiff Files Analyzed* and *DMD Modes*. The frequencies are the x-value corresponding to the highest y-value in the bottom two left graphs in Figures 16-19.

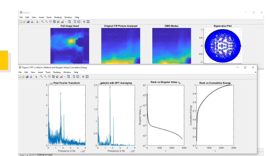

Figure 18: Frequency of Detonation Waves Tiff Set From Exhaust View 1

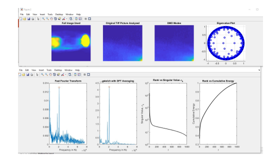

Figure 19: Frequency of Detonation Waves Tiff Set From Exhaust View 2

The exhaust views from Figures 18 and 19, like mentioned before, do not show the detonation waves like in Figures 16 and 17 and are cropped at specific dimensions to attain these frequencies. The FFT's of both Figures also show more interference compared to Figures 16 and 17 because this analysis looks at the entire exhaust. Finally, in Figure 19 specifically, the frequency shown based on FFT and Welch's method is technically an interference since Figure 18 shows the real frequency. That frequency can be found as the second highest amplitude in the frequency domains but finding the correct crop will take more time than is necessary.

Future Work

This will lead to finding the frequencies of detonation waves at the rear view of the RDE but with a nozzle attached to the back of it to attempt to reduce the detonation wave frequency at that end compared to the front end.

References

1. Lu F. K. and Braun E. M. (2019). Journal of Propulsion and Power. *Rotating Detonation Wave Propulsion: Experimental Challenges, Modeling, and Engine Concepts*. *30*(5), 1125-1139.
2. *Technology Development Status of Pressure Gain Combustion for Power Generating Gas Turbines:* EPRI, Palo Alto, CA: 2019. 3002015798.
3. Braun, E., Dunn, N., & Lu, F. (2010). Testing of a Continuous Detonation Wave Engine with Swirled Injection. *48th AIAA Aerospace Sciences Meeting Including the New Horizons Forum and Aerospace Exposition*. https://doi.org/10.2514/6.2010-146
4. Kailasanath, K. (2011). The Rotating Detonation-Wave Engine Concept: A Brief Status Report. *49th AIAA Aerospace Sciences Meeting Including the New Horizons Forum and Aerospace Exposition*. https://doi.org/10.2514/6.2011-580
5. Bennewitz, J. W., Bigler, B. R., Hargus, W. A., Danczyk, S. A., and Smith, R. D. "Characterization of detonation wave propagation in a rotating detonation rocket engine using direct high-speed imaging," *2018 Joint Propulsion Conference*. 2018.
6. Brunton, S. L., & Kutz, J. N. (2020). Data-driven science and engineering: machine learning, dynamical systems and control. Cambridge University Press.
7. Cohen M. [Mike X Cohen] (2019, December 20). *Welch's method for smooth spectral decomposition*. [Video]. YouTube. https://www.youtube.com/watch?v=YK1F0-3VvQI
8. Cohen M. [Mike X Cohen] (2019, December 20). *Welch's method for smooth spectral decomposition*. [Video]. YouTube. https://www.youtube.com/watch?v=YK1F0-3VvQI
9. Robertson N. (2019, January 13). *Use Matlab Function pwelch to Find Power Spectral Density – or Do It Yourself*. Use Matlab Function pwelch to Find Power Spectral Density – or Do It Yourself - Neil Robertson (dsprelated.com)

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